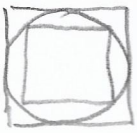
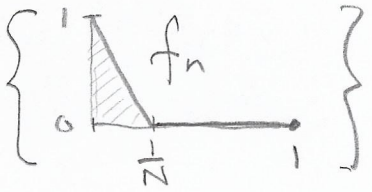
Homework #7

1a. No. Only in finite-dimensional spaces are all norms equivalent.



b. 

$$\lim_{N\to\infty} \int_0^1 |f_n| = 0$$

$$\max_{[0,1]} f_n = 1 \quad \forall n$$

Ames 1-3-③

2. for HW7 Example 2 1D isentropic flow of perfect gas

$u_t + uu_x + \frac{1}{\rho}p_x = 0$ momentum balance

$\rho_t + \rho u_x + up_x = 0$ continuity

u = velocity, p = pressure, ρ = density

c = vel. of sound, γ = ratio of specific heats

$p\rho^{-\gamma} = \alpha$ const , $c^2 = \frac{dp}{d\rho}$

Eliminate the pressure & write 2 eqns for u & ρ.

No: If $\frac{dp}{d\rho} = c^2$ (not const.), then $\boxed{p = c^2\rho + \kappa}$. But $p|_{\rho=0}$ must $=0$, so $\kappa = 0$.

$p = \alpha\rho^\gamma$, c is a local speed.

$$u_t + uu_x + \frac{1}{\rho}\alpha\gamma\rho^{\gamma-1}\rho_x = \boxed{u_t + uu_x + \alpha\gamma\rho^{\gamma-2}\rho_x = 0}$$

$$\boxed{\rho_t + \rho u_x + up_x = 0}$$

(u ρ)

$$A = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ 1 & u & 0 & \alpha\gamma\rho^{\gamma-2} \\ a_2 & b_2 & c_2 & d_2 \\ 0 & \rho & 1 & u \\ 1 & y' & 0 & 0 \\ 0 & 0 & 1 & y' \end{bmatrix}$$

$$\text{discr}A = (1\cdot u - 0\cdot 0 + u\cdot 1 - \rho\cdot 0)^2 = 4u^2$$

$$- 4(1\cdot 1 - 0\cdot 0)(u\cdot u - \rho\alpha\gamma\rho^{\gamma-2}) \quad -4(u^2 - \alpha\gamma\rho^{\gamma-1})$$