

Day - 6 ⁽²³⁾ Outline 4/20/21

Poisson/Laplace by FD cont'd (5 methods!)

Direct solution

- naive solution - slow
- solution recognizing banded structure

Iterative solution

- Jacobi iteration
- Gauss-Seidel
- SOR

Trying to solve $Ax = b$

Write $A = L + D + U$

↑ strictly lower tri ↑ diagonal ↑ strictly upper tri

$$(L + D + U)x = b$$

$$Dx = b - (L + U)x$$

$$Dx_{k+1} = b - (L + U)x_k \quad (\text{Jacobi})$$

A fixed point of this iteration is a solution of $Ax = b$.

$$x_{k+1} = \underbrace{D^{-1}b}_{\equiv C} - \underbrace{D^{-1}(L+U)}_{\equiv T} x_k \quad \text{Jacobi}$$

$$(2) \quad \boxed{x_{k+1} = Tx_k + C} \quad \text{an affine iteration}$$

What is important for convergence?

Answer: T

because if x^* is a fixed point

and define $y_k = x_k - x^*$

if you plug this into (2) you will find

$$\boxed{y_{k+1} = Ty_k} \quad \text{linear iteration}$$

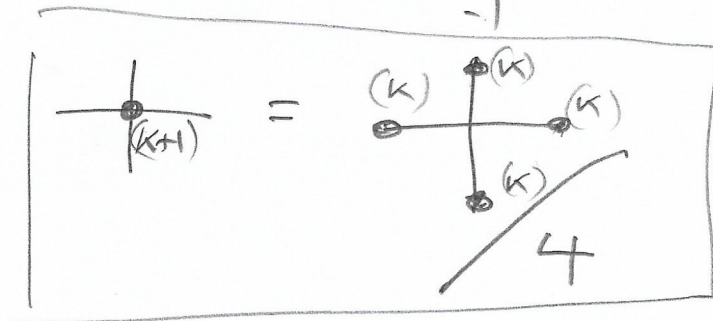
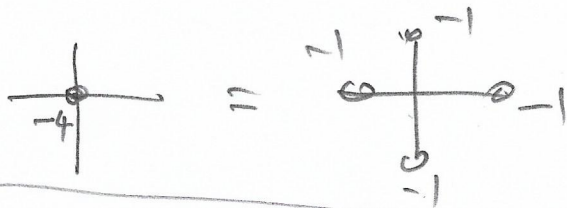
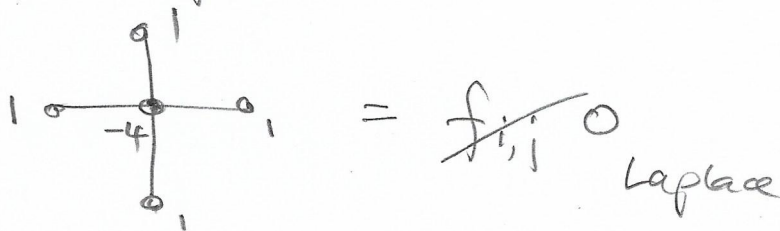
Therefore this converges if $\rho(T) = \max_i |\lambda_i| < 1$

↑
Spectral radius

↑
Eigenvalues of T

Spectrum of T is key.

For our problem



Jacobi iteration