538 Day 4 2/11/21

Method

$$y(t_{k+1}) = y(t_k) + h\,\Phi(t_k, y_k, h)$$

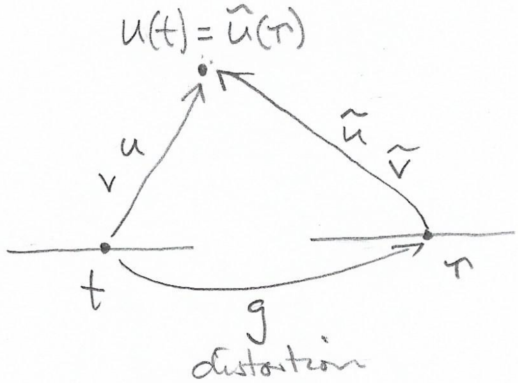
defines method

Euler $\Phi(t_k, y_k, h) = f(t_k, y_k)$.

A non-autonomous ODE for testing
by distorting time in

2/11/21

$$\frac{d\hat{u}}{d\tau} = -\hat{v} \,, \qquad \frac{d\hat{v}}{dt} = \hat{u} \,, \qquad \begin{bmatrix} \hat{u}(0) \\ \hat{v}(0) \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$



$$\frac{du(t)}{dt} = \frac{d\hat{u}(g(t))}{dt} = \hat{u}'(g(t)) \cdot g'(t)$$

With $g(t) = t - \dfrac{\sin t}{1.2}$ (arbitrarily)

$$g'(t) = 1 - \frac{\cos t}{1.2}$$

we have

$$u'(t) = -\hat{v}(g(t)) \cdot \left(1 - \frac{\cos t}{1.2}\right)$$

$$\boxed{u'(t) = -v(t) \cdot \left(1 - \frac{\cos t}{1.2}\right)}$$

And similarly

$$\boxed{v'(t) = u(t)\left(1 - \frac{\cos t}{1.2}\right)}$$

Exact solution is

$$u(t) = \hat{u}(g(t)) = 3\cos\left(t - \frac{\sin t}{1.2}\right)$$

$$v(t) = \hat{v}(g(t)) = 3\sin\left(t - \frac{\sin t}{1.2}\right).$$

$$\begin{bmatrix} u \\ v \end{bmatrix}\Bigg|_{t=2\pi} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{u} \\ \hat{v} \end{bmatrix}\Bigg|_{\tau=2\pi}$$