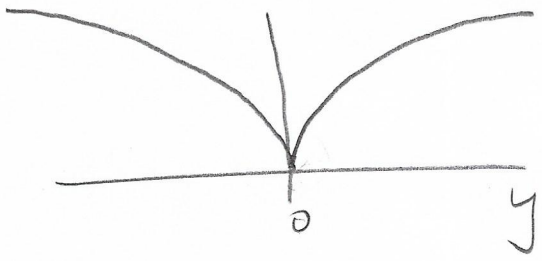
Can use any norm on $\mathbb{R}^n$:

e.g. $\|y\|_2 = \sqrt{y_1^2 + y_2^2 + \cdots y_n^2}$

$\|y\|_\infty = \max_i |y_i|$

Example of non-L function: leaky bucket

$$y' = \underbrace{-|y|^{\frac{1}{2}}}_{f(y)}$$



Euler

$$y_0 = y(0) \text{ given}$$

$$y_{k+1} = y_k + h f(t_k, y_k) \qquad k = 0, 1, 2, \ldots m$$

h: step size in t

Example

$$y'(t) = t + y, \quad t \in [1, 5]$$
$$y(1) = 2$$

Exact solution:

$$y(t) = \frac{4}{e} e^t - t - 1$$

Check:

$$\text{LHS} = \frac{dy(t)}{dt} = \frac{4}{e} e^t - 1 + 0$$

$$\text{RHS} = t + \frac{4}{e} e^t - t - 1$$

✓ (1)

An example of an IVP with $n > 1$

$$u' = -v \qquad u(0) = 3$$
$$v' = u \qquad v(1) = 0$$

$$y = \begin{bmatrix} u \\ v \end{bmatrix} \in \mathbb{R}^2 \qquad y(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$f(t, y) = \begin{bmatrix} -v \\ u \end{bmatrix}$$

Exact solution

$$\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = 3 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$